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TRIGONOMETRÍA

DIRIGIDA

SEMANA 11

PROBLEMA 1

Si $\theta \in IVC$ y $E=|\sec\theta+1|+|\sec\theta-1|$, determine la variación de la expresión E.

A) $\left\langle \frac{9}{2}; 10 \right\rangle$ B) $\left\langle \frac{\sqrt{3}}{5}; \frac{2}{5} \right\rangle$ C) $\langle 1; 2 \rangle$

D) $\left\langle \frac{3}{5}; \frac{7}{5} \right\rangle$ E) $\langle 2; +\infty \rangle$

$|a| = a$ (+)

$|a| = -a$ (−)

Resolución

$E = |\underbrace{\sec\theta+1}_{+}| + |\underbrace{\sec\theta-1}_{+}|$; $\theta \in IVC \rightarrow \sec\theta > 1$

$E = \sec\theta + 1 + \sec\theta - 1$

$E = 2\sec\theta$; $\theta \in IVC$

$\sec\theta > 1 \rightarrow \underbrace{2\sec\theta}_{E} > 2$

$\therefore E \in \langle 2; +\infty \rangle$

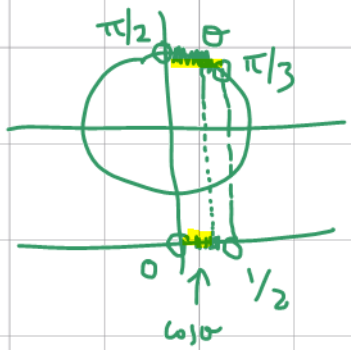
PROBLEMA 2

Si $\frac{2\cos\theta-1}{\cos\theta-1}>0;\ \theta\in\left\langle 0;\frac{\pi}{2}\right\rangle$, calcule la extensión de $\csc^2\theta$.

A) $\left\langle \frac{5}{2};+\infty\right\rangle$ B) $\langle 1;+\infty\rangle$ C) $\left\langle \frac{3}{2};+\infty\right\rangle$

D) $\left\langle \frac{4}{3};+\infty\right\rangle$ E) $\left\langle 1;\frac{4}{3}\right\rangle$

$0<\cos\theta<\frac{1}{2} \Rightarrow \frac{\pi}{3}<\theta<\frac{\pi}{2}$



Resolución

$\frac{2\cos\theta-1}{\cos\theta-1}>0$; $0<\theta<\frac{\pi}{2}$ → $0<\cos\theta<1$

$-1<\cos\theta-1<0$

→ $2\cos\theta-1<0$

→ $\cos\theta<\frac{1}{2}$

Pero $0<\cos\theta<1$

$0<\cos\theta<\frac{1}{2}$

$0<\cos^2\theta<\frac{1}{4} \rightarrow 0<1-\sen^2\theta<\frac{1}{4}$

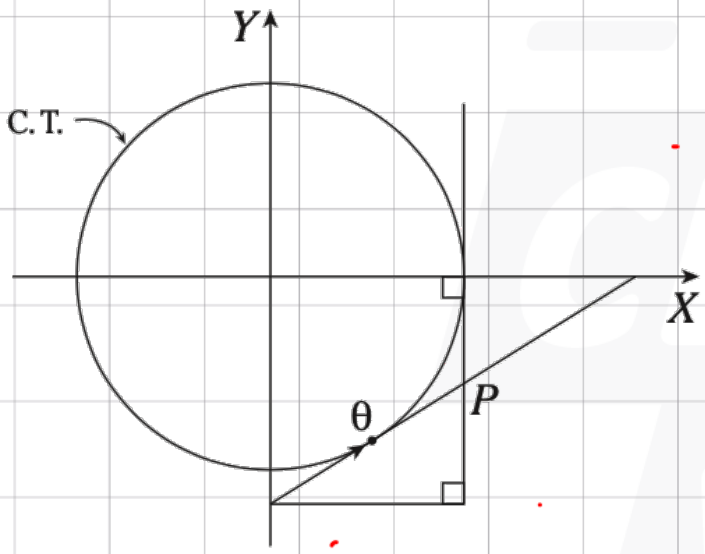
$-1<-\sen^2\theta<-\frac{3}{4} \rightarrow 1>\sen^2\theta>\frac{3}{4}$

$1<\frac{1}{\sen^2\theta}<\frac{4}{3} \rightarrow 1<\csc^2\theta<\frac{4}{3}$

$\therefore\ \csc^2\theta\in\left\langle 1;\frac{4}{3}\right\rangle$

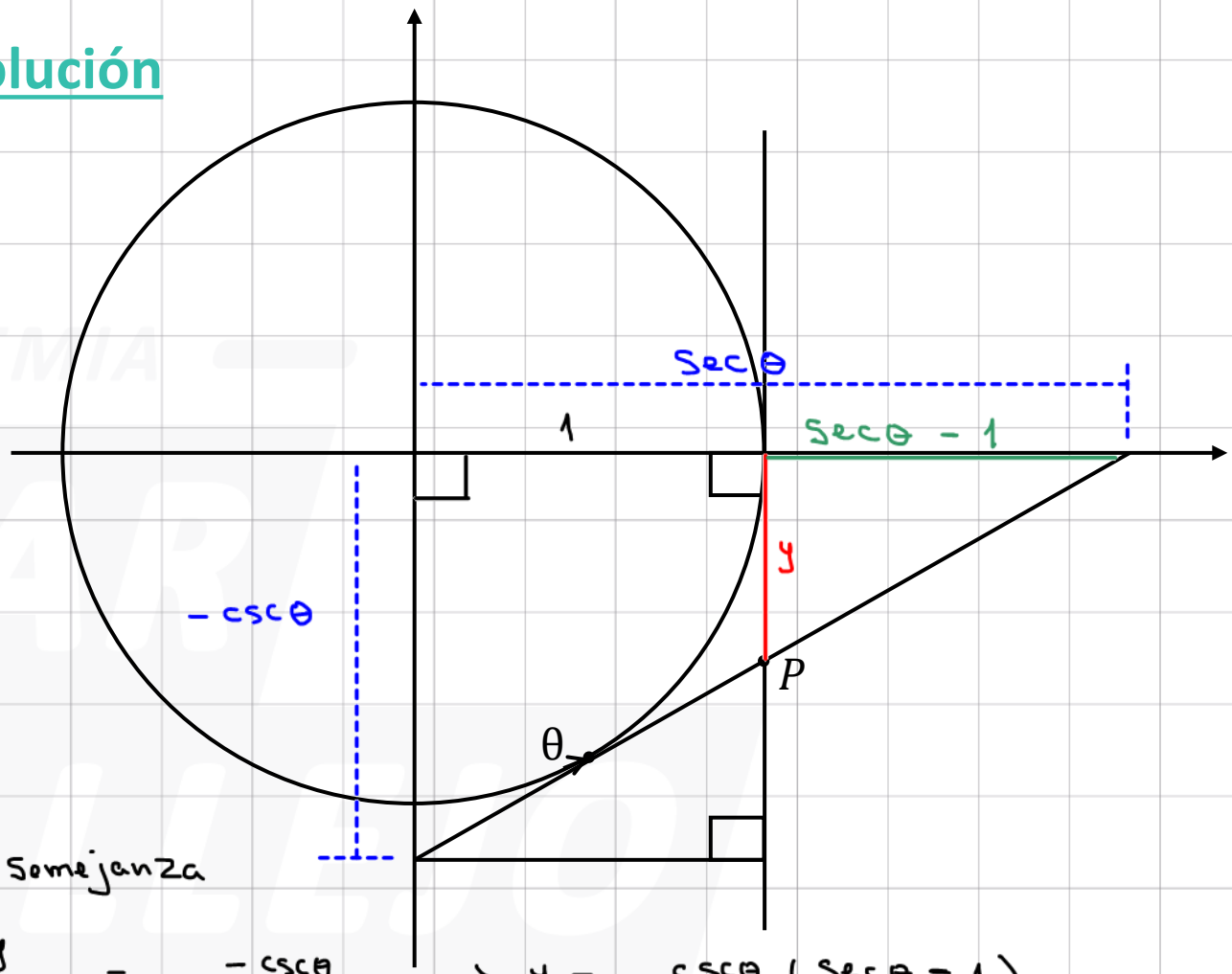
PROBLEMA 3

Calcule la ordenada del punto P en términos de θ.



A) $\cot\theta - \csc\theta$
B) $\tan\theta - \sec\theta$
C) $\sec\theta - \tan\theta$
D) $\csc\theta - \tan\theta$
E) $\csc\theta - \cot\theta$

Resolución



Por semejanza

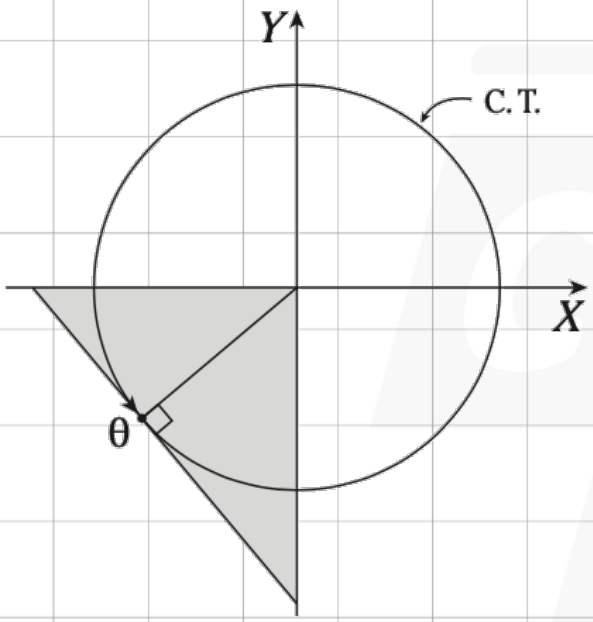
$$\frac{y}{\sec\theta - 1} = \frac{-\csc\theta}{\sec\theta} \rightarrow y = -\frac{\csc\theta}{\sec\theta}(\sec\theta - 1)$$

$$y = -\csc\theta + \cot\theta$$

$$\therefore \quad -y = \csc\theta - \cot\theta$$

PROBLEMA 4

Si el área de la región sombreada es 8/3, calcule sen2θ.

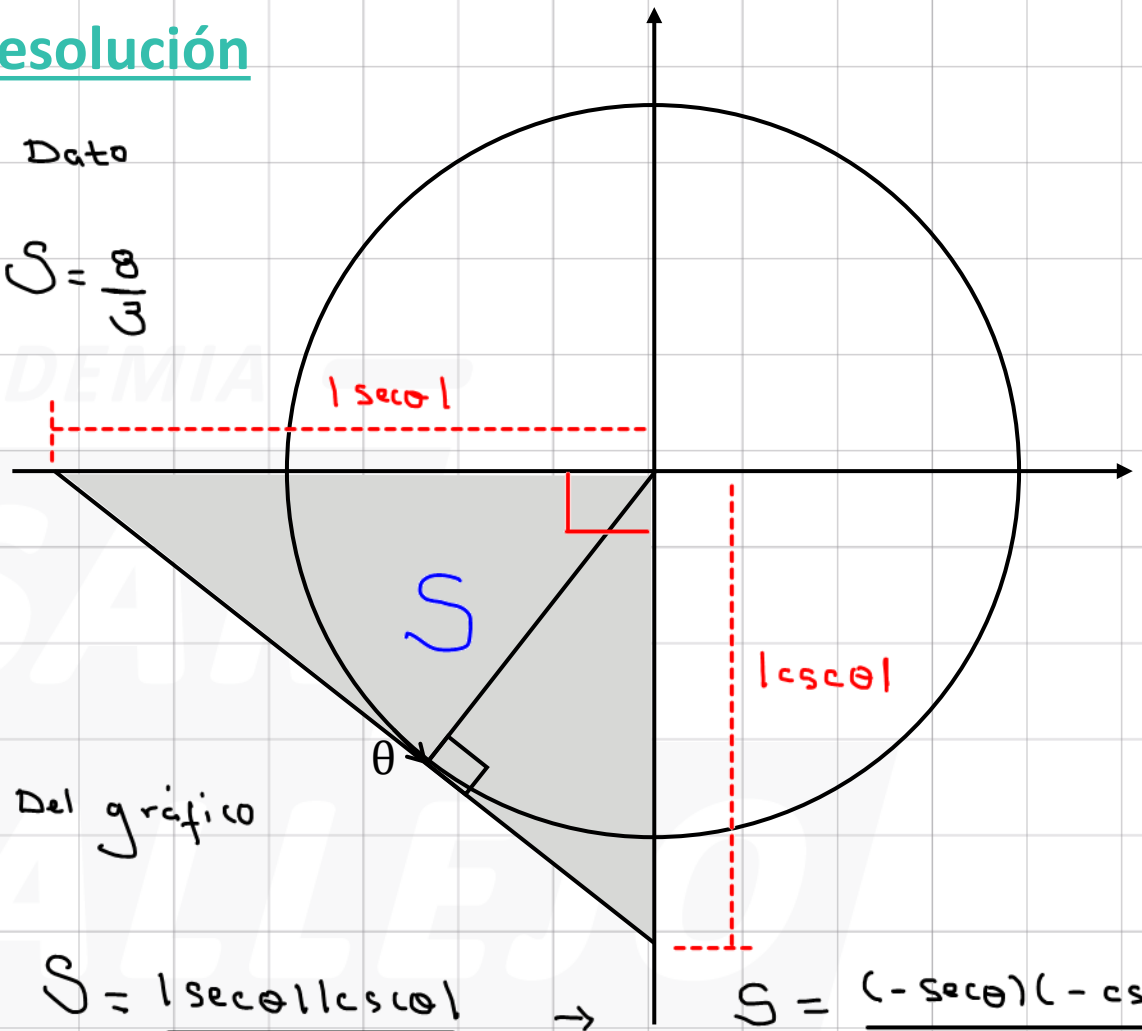


A) 3/4 B) 2/3 C) 3/8
D) 1/2 E) 1/4

Resolución

Dato

$S = \frac{8}{3}$



Del gráfico

$S = \frac{|\sec\theta||\csc\theta|}{2} \rightarrow S = \frac{(-\sec\theta)(-\csc\theta)}{2}$

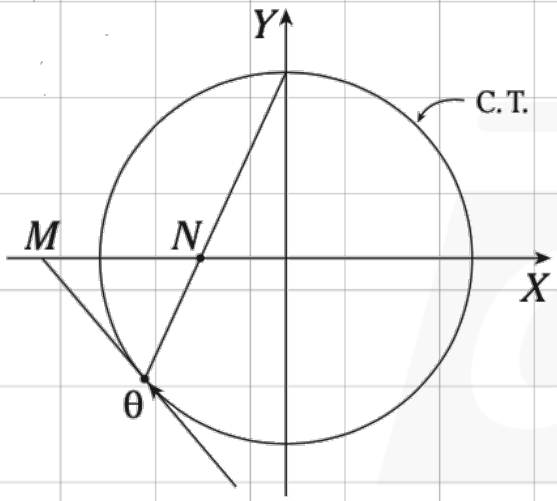
$S = \frac{\sec\theta\csc\theta}{2} \rightarrow \frac{8}{3} = \frac{1}{2\,\text{sen}\theta\cos\theta}$

$\frac{8}{3} = \frac{1}{\text{sen}2\theta}$

$\therefore \text{sen}2\theta = 3/8$

PROBLEMA 5

Del gráfico, calcule MN en términos de θ.



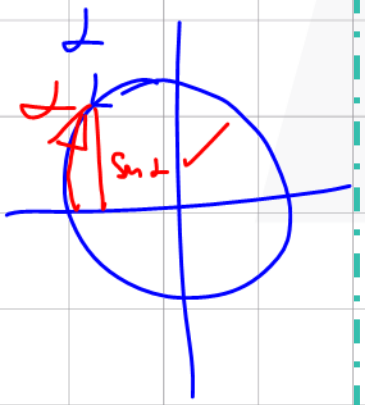
A) $\dfrac{\sec\theta-\cos\theta+\tan\theta}{\operatorname{sen}\theta-1}$

B) $\dfrac{\sec\theta+\cos\theta-\tan\theta}{\operatorname{sen}\theta-1}$

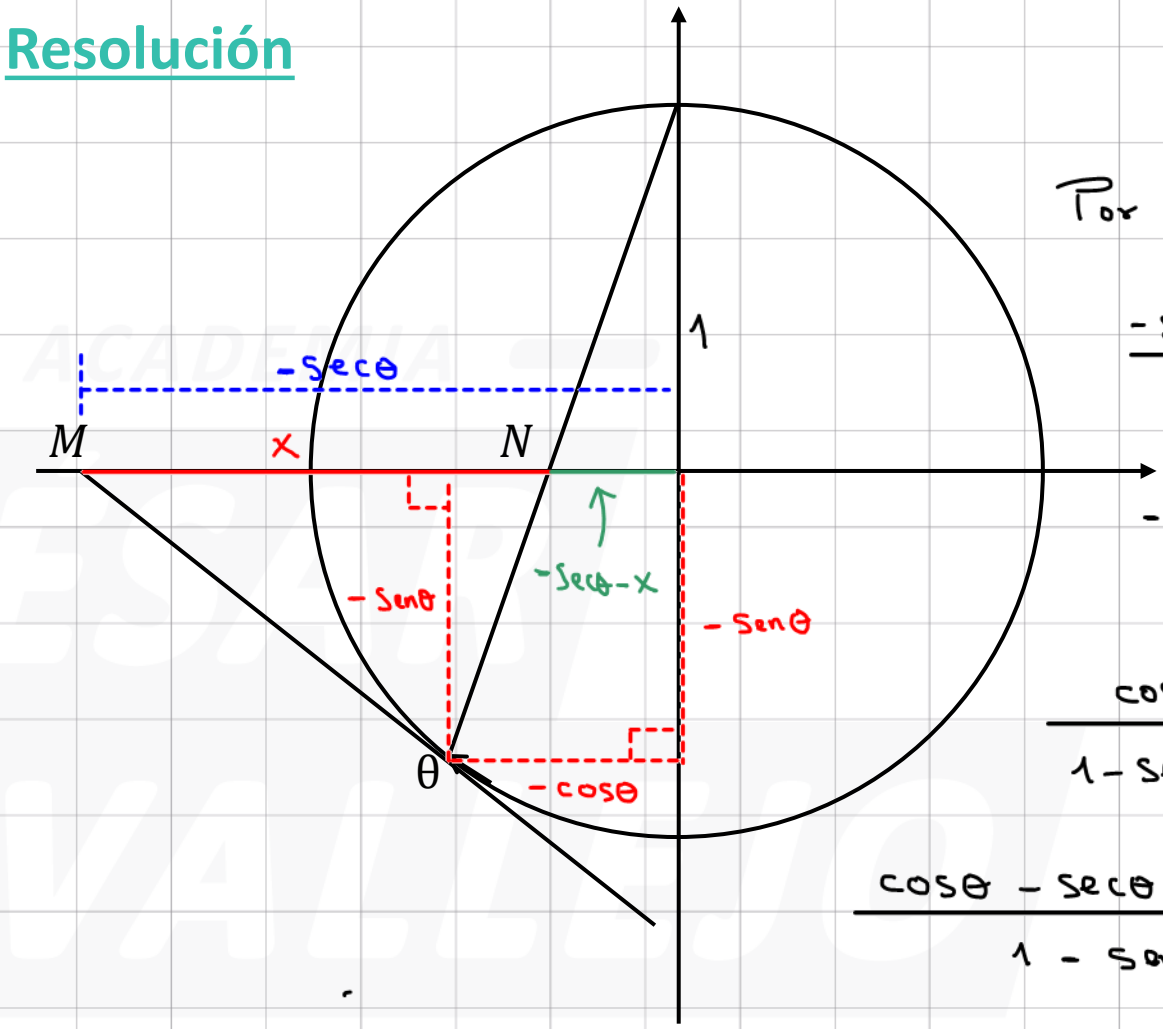
C) $\dfrac{\sec\theta-\operatorname{sen}\theta\cos\theta}{\operatorname{sen}\theta+1}$

D) $\dfrac{\sec\theta-\cos\theta-\tan\theta}{\operatorname{sen}\theta-1}$

E) $\dfrac{\sec\theta+\cos\theta+\tan\theta}{\operatorname{sen}\theta+1}$



Resolución



Por semejanza

$$\frac{-\sec\theta-x}{1}=\frac{-\cos\theta}{1-\operatorname{sen}\theta}$$

$$-\sec\theta-x=\frac{-\cos\theta}{1-\operatorname{sen}\theta}$$

$$\frac{\cos\theta}{1-\operatorname{sen}\theta}-\sec\theta=x$$

$$\frac{\cos\theta-\sec\theta+\tan\theta}{1-\operatorname{sen}\theta}=x$$

$$x=\frac{(-1)(\cos\theta-\sec\theta+\tan\theta)}{(-1)(1-\operatorname{sen}\theta)}$$

$$\therefore\ x=\frac{\sec\theta-\cos\theta-\tan\theta}{\operatorname{sen}\theta-1}$$

PROBLEMA 6

Si $\csc\theta = 1+\tan\alpha$, calcule la variación de θ.

Considere que $\alpha \in \left\langle \pi; \frac{5\pi}{4} \right\rangle$ y $\theta \in \langle 0; 2\pi \rangle$.

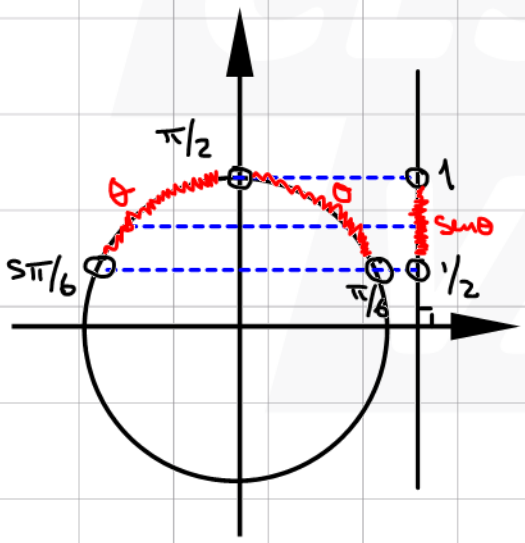
A) $\left\langle \frac{\pi}{4}; \frac{3\pi}{4} \right\rangle$

B) $\left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle$

C) $\left\langle \frac{\pi}{3}; \frac{2\pi}{3} \right\rangle$

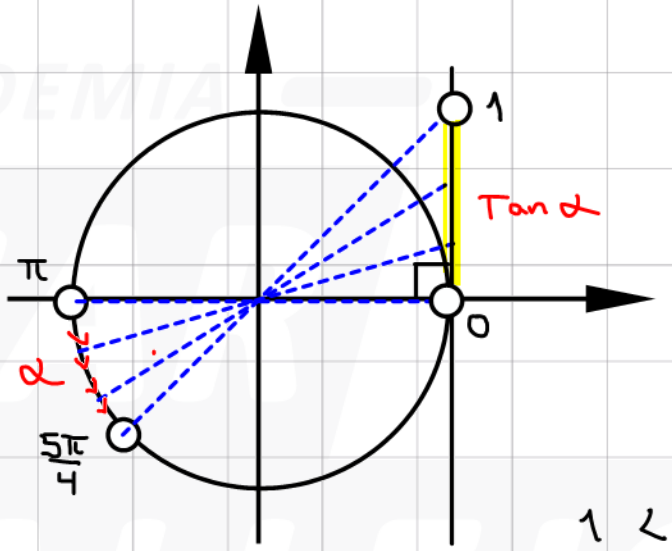
D) $\left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle - \left\{ \frac{\pi}{2} \right\}$

E) $\left\langle \frac{3\pi}{4}; \pi \right\rangle$



Resolución

$\csc\theta = 1 + \tan\alpha \; ; \; \pi < \alpha < \frac{5\pi}{4} \; ; \; 0 < \theta < 2\pi$



Del gráfico

$0 < \tan\alpha < 1$

$1 < 1 + \tan\alpha < 2$

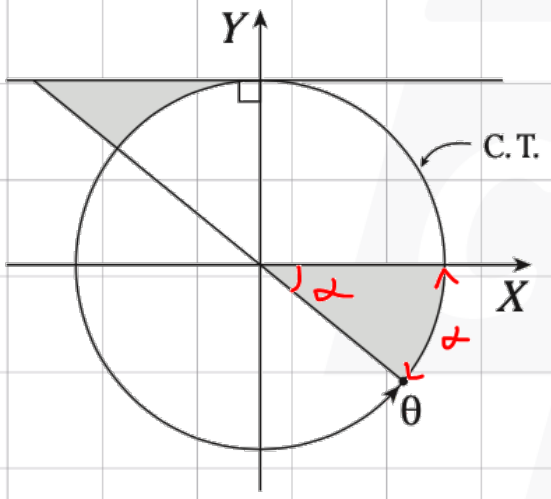
$1 < \csc\theta < 2$

$1 > \frac{1}{\csc\theta} > \frac{1}{2} \rightarrow 1 > \operatorname{Sen}\theta > \frac{1}{2}$

$\therefore \theta \in \left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle - \left\{ \frac{\pi}{2} \right\}$

PROBLEMA 7

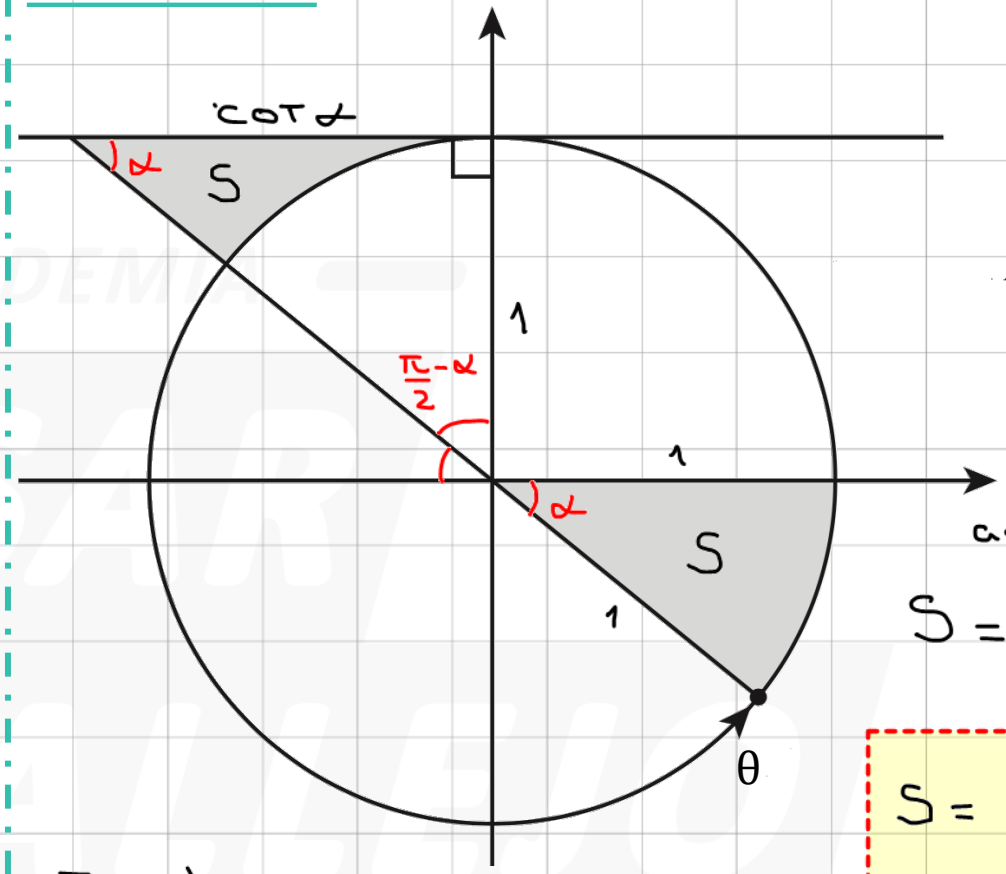
Si las regiones sombreadas son equivalentes, calcule $\csc(\cot\theta)+\operatorname{sen}(\cot\theta)$.



A) 4 B) −4 C) $\frac{10}{3}$

D) 2 E) −2

Resolución



$$S=\frac{\alpha\cdot 1^2}{2} \rightarrow \boxed{S=\frac{\alpha}{2}}$$

además

$$S=\frac{1\cdot \cot\alpha}{2}-\frac{(\frac{\pi}{2}-\alpha)\cdot 1^2}{2}$$

$$\boxed{S=\frac{\cot\alpha}{2}-\frac{\pi}{4}+\frac{\alpha}{2}}$$

Igualamos

$$\frac{\cot\alpha}{2}-\frac{\pi}{4}+\frac{\alpha}{2}=\frac{\alpha}{2} \rightarrow \cot\alpha=\frac{\pi}{2}$$

$\theta+\alpha=2\pi$, $\alpha=2\pi-\theta$

$$\rightarrow \cot\theta=-\frac{\pi}{2}$$

Piden

$$\csc(\cot\theta)+\operatorname{sen}(\cot\theta)=\underbrace{\csc(-\tfrac{\pi}{2})}_{-1}+\underbrace{\operatorname{sen}(-\tfrac{\pi}{2})}_{-1}=-2$$

Problema 1

Resolución

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